Connecticut Common Core Algebra 1 Curriculum

Professional Development Materials

Unit 2 Equations and Inequalities

Contents

Activity 2.1.2 Representing Expressions with Stories and Flowcharts

Activity 2.1.3 Representing Expressions with Algebra Arrows

Activity 2.2.1 Solving Linear Equations using Flowcharts

Activity 2.2.2 Solving Linear Equations with Algebra Tiles

Activity 2.2.6 Station Problems

Activity 2.3.1 Combining Like Terms with Algebra Tiles

Activity 2.3.3 Solving Equations with Variables on Both Sides

Activity 2.3.5 Solving Equations with Balance Scales

Activity 2.4.2 Distributive Property with Algebra Tiles

Activity 2.4.4 Walk-A-Thon

Activity 2.4.5 Epic Win Epic Fail

Activity 2.4.6 Transforming Literal Equations with Flow Charts

Unit 2 Investigation 5 Overview: Formulas and Literal Equations

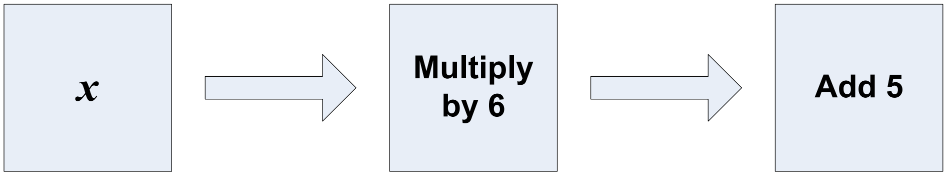
Unit 2 Investigation 6 Overview: Linear Inequalities

Representing Expressions with Stories and Flowcharts

When we think of stories we usually don't think of algebra, but mathematical expressions tell stories too! When we see an expression involving a variable, something is happening to that variable. In other words, something is *being done* to the variable. Let's first use a story to represent what is being done to the variable. Use the order of operations to decide what steps are taken to evaluate the expression.

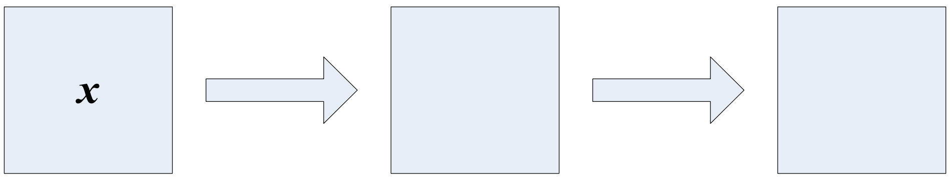
Expression	Story of x
$x + 6$	
$x/3$	
$8x$	
$x - 5$	
$2x + 6$	
$-6x + 3$	
$\frac{x}{2} - 4$	
$\frac{x + 5}{2}$	
$8 - x$	
$\frac{-7x + 2}{8}$	
$\frac{2x + 7}{3}$	
$\frac{3 - x}{5}$	

We can also represent the *story of x* by a *flowchart*. The flowchart below displays the story of x for the expression $6x + 5$.

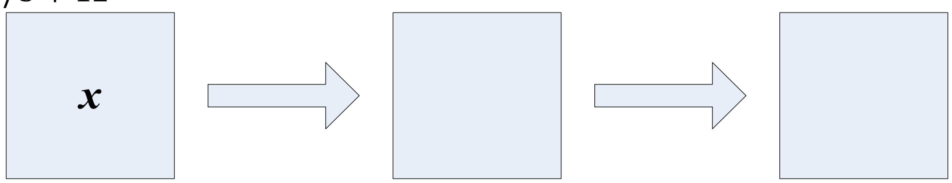


1. Use a flowchart to represent the following expressions involving two operations:

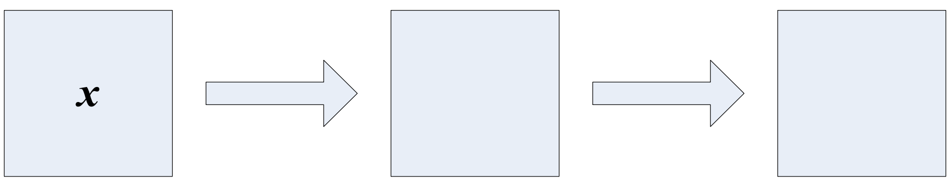
A. $7x + 1.75$



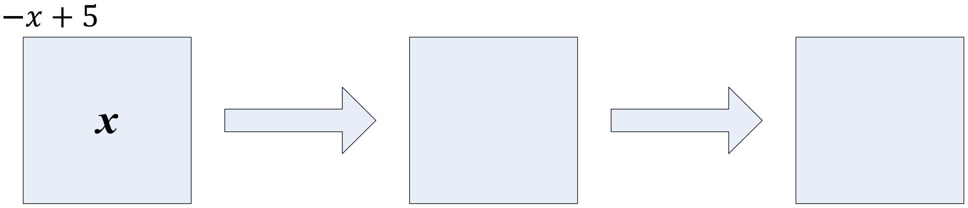
B. $x/3 + 12$



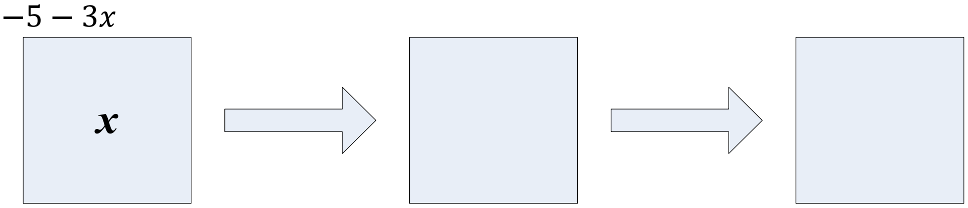
C. $5x - 11$



D. $-x + 5$

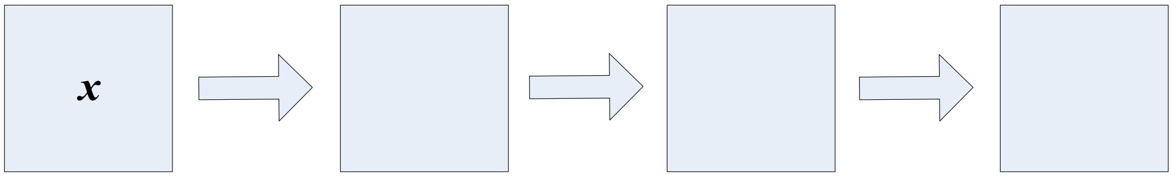


E. $-5 - 3x$

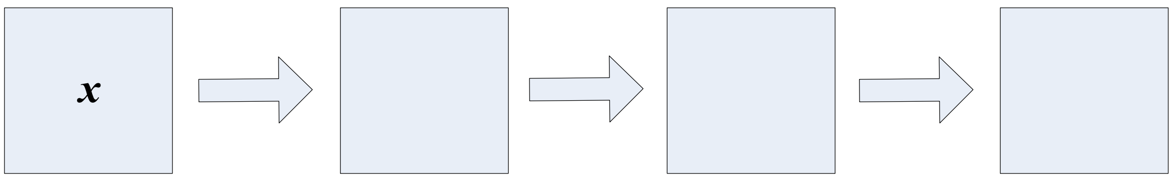


2. Use a flowchart to represent the following expressions involving three operations:

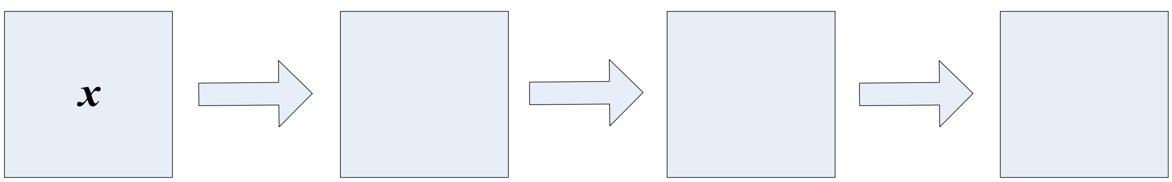
A. $\dfrac{2x+5}{6}$



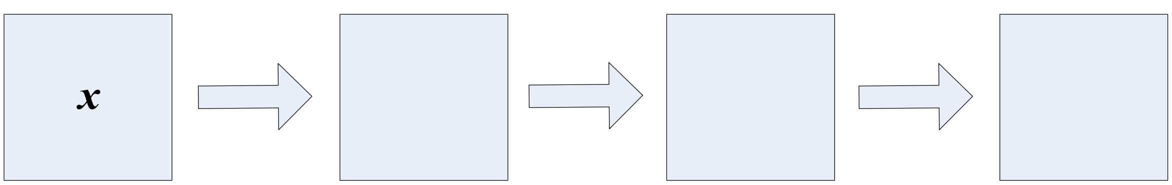
B. $\dfrac{-3x+5}{11}$



C. $\dfrac{4x-0.5}{-3}$



D. $\dfrac{6-2x}{5}$



3. Let's now convert stories to mathematical expressions. Given the story on x, write a mathematical expression that describes the story. Use x to represent the unknown number.

   A. Multiply a number by 7.

   B. Add 14 to a number.

   C. Subtract a number from 12.

   D. Subtract 6 from a number.

   E. Divide a number by -3

   F. Divide -8 by a number.

   G. Multiple a number by 4, then add 3.

   H. Multiply a number by 8, then add –11.

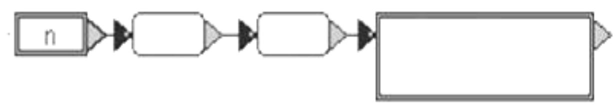
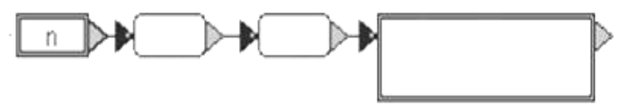
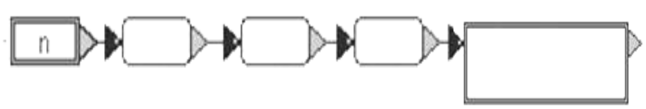
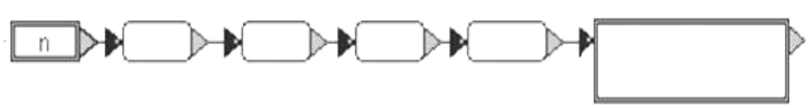
   I. Subtract 4 from a number, then multiply by 6.

   J. Add -1 to a number, then divide by 3.

   K. Divide a number by 2, then subtract 13.

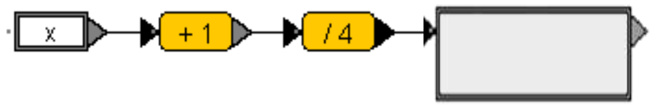
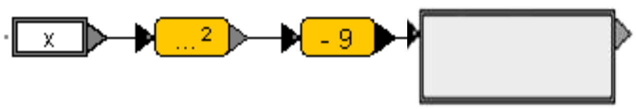
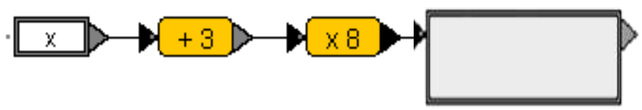
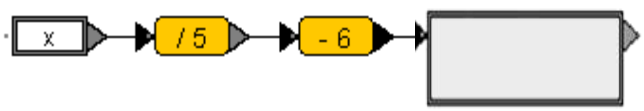
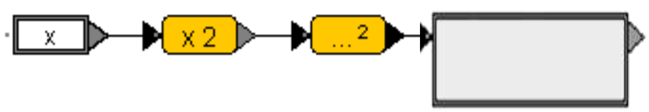
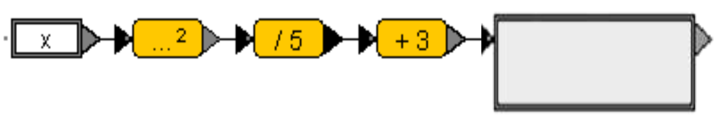
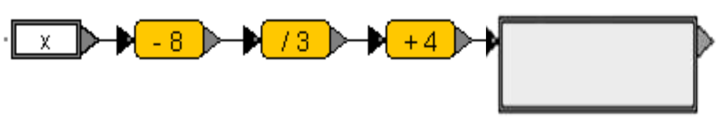
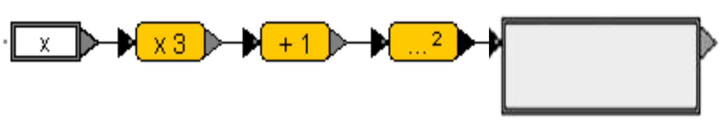
Representing Expressions with Algebra Arrows

Write the "story of *n*" for each of the following algebraic expressions. Carefully identify the order in which the operations occur on the variable term. Then build each story using the *Algebra Arrows* applet on the Freudenthal Institute website*. Sketch a chain of arrows for each expression and evaluate each expression when $n = 3$.

Expression	**Story of *n***	**Algebra Arrows Chain**	**Evaluate when $n = 3$**
$3n+7$			
$\frac{n}{3}-10$			
$2(n-5)$			
$\frac{n-8}{4}$			
$7(n^2-1)$			
$6n^2+2$			
$(n+3)^2-6$			
$\frac{3n^2+2}{5}$			

* http://www.fi.uu.nl/wisweb/en/

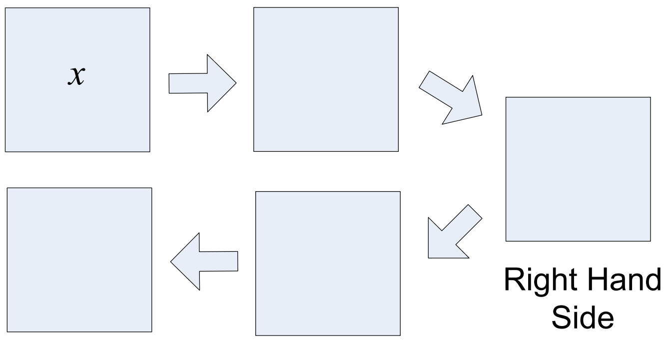
Fill in the expression created by each algebra arrow chain, and then tell the story of *x*. Then evaluate the expression when $x = 5$.

Algebra Arrows Chain	**Story of *x***	**Evaluate when $x = 5$**
		
		
		
		
		
		
		
		
		

Solving Linear Equations using Flowcharts

You will now use flowcharts to solve one-step and two-step linear equations. Apply the corresponding steps to the equation on the right side. Check your solution.

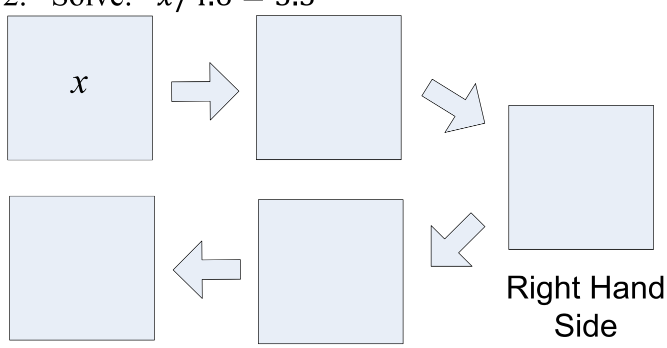
1. Solve $x - 5.4 = 19.8$



$x - 5.4 = 19.8$

Check:

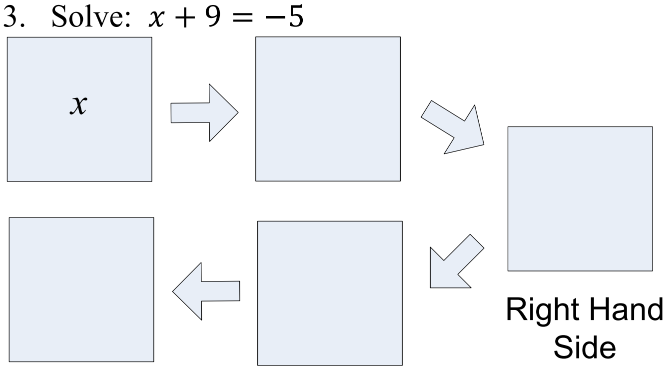
2. Solve: $x/4.6 = 3.5$



$x/4.6 = 3.5$

Check:

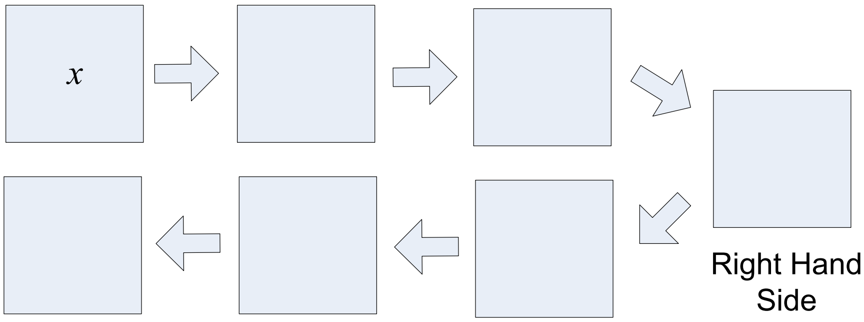
3. Solve: $x + 9 = -5$



$x + 9 = -5$

Check:

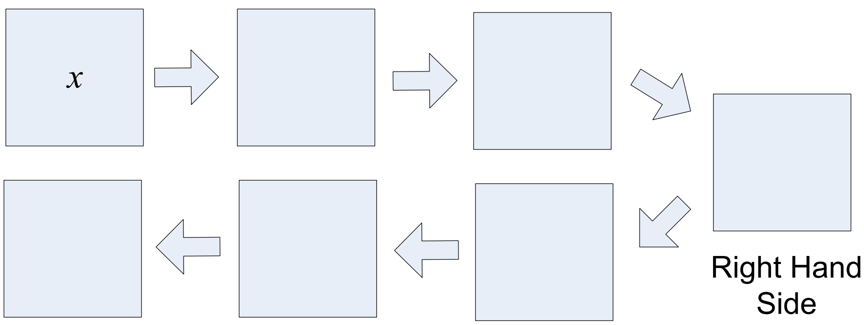
4. Solve $3x + 4 = -11$



$3x + 4 = -11$

Check:

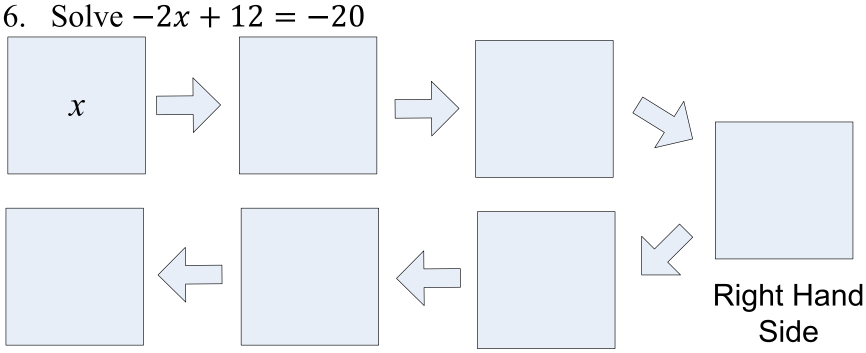
5. Solve $2x - 8 = 14$



$2x - 8 = 14$

Check:

6. Solve $-2x + 12 = -20$



$-2x + 12 = -20$

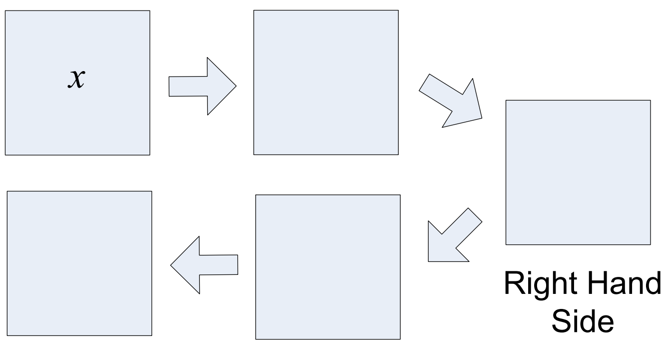
Check:

Model the situations below with a linear equation. For each problem, identify the unknown, create an equation, solve the equation using a flowchart, and then check your solution.

7. Kevin bought seven tickets to the Haunted Graveyard at Lake Compounce for $209.93. How much does one ticket cost?

   Identify the unknown:

   Equation:

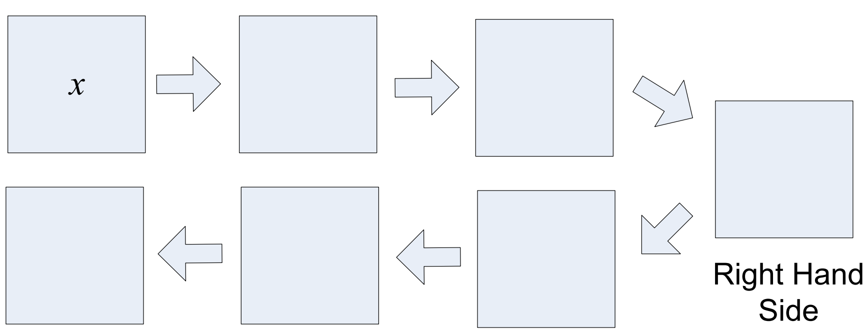


Check:

8. Verizon charges $18.75 per month for phone service and $0.08 per minute. Last month my bill was $33.63. How many minutes did I use?

   Identify the unknown:

   Equation:

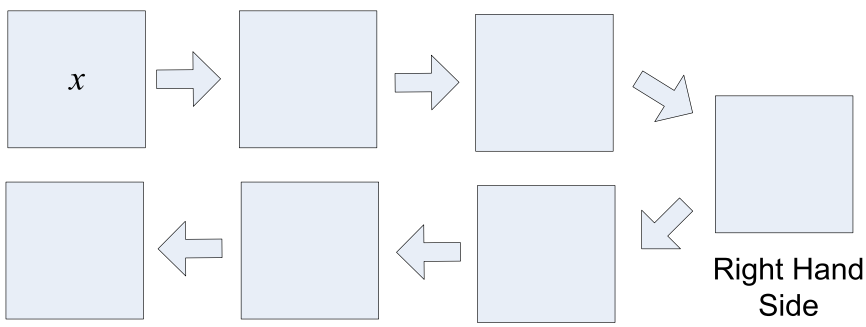


Check:

9. Jose spent $177.69 of his birthday money. He bought an iPod for $159 and 21 songs from iTunes. How much did each song cost?

   Identify the unknown:

   Equation:

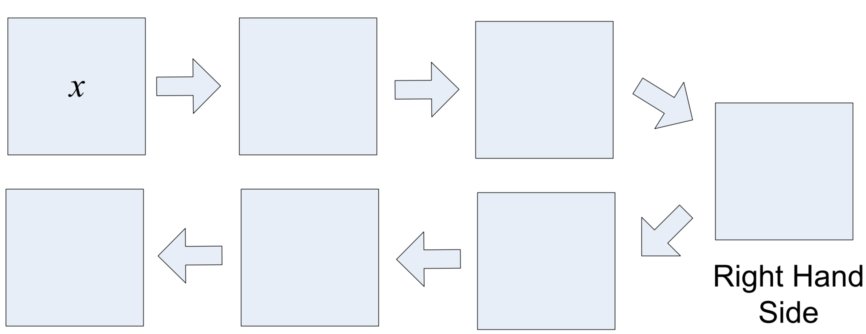


Check:

10. Your school band needs to buy new recording equipment. The equipment will cost $3000. The band has collected $1200 from previous fundraisers. If the band sells sandwiches at $5 each, how many sandwiches must they to sell to raise the remaining funds?

    Identify the unknown:

    Equation:



Check:

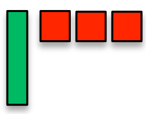
Solving Linear Equations with Algebra Tiles

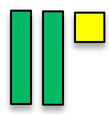
KEY:

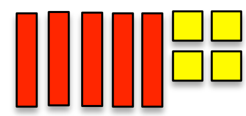
 = negative number  = negative variable

 = positive number  = positive variable

Write the mathematical expression represented by each group of tiles.

1. 

2. 

3. 

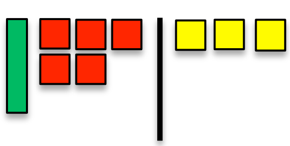
Model each expression below with your tiles. Then, draw a picture of what your tiles look like.

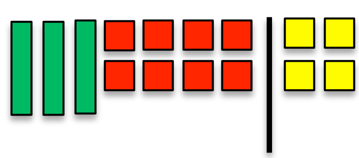
4. $x + 1$

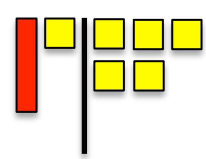
5. $2b - 3$

6. $3g - 5$

Write the equation represented by each group of tiles.

7. 

8. 

9. 

Model each equation with your tiles. Then, draw a picture of what your tiles look like below.

10. $-2j + 1 = 11$

11. $-6 = 4 - 5x$

12. $7 = 2w - 1$

Model each equation with your algebra tiles. Then solve each equation by making zero pairs and rearranging the groups as necessary. Check your solutions.

13. $x + 5 = -2$

14. $-3h - 1 = 11$

15. Yellow Cab Company charges $7 for picking you up and $2 per mile. How many miles can you ride if you have $20?

Identify the unknown:

Equation:

16. Mary has 6 CD's. If she has 7 less than John, how many CD's does John have?

Identify the unknown:

Equation:

STATION 1 PROBLEM

$$3 - 2m = -21$$

Answer to Another Station

–20

STATION 2 PROBLEM

Better Buy sells laptops that had 4GB of memory for $495. You can buy additional memory for $97 per GB. If you have $980 to buy a laptop and additional memory, how much additional memory can you buy?

Answer to Another Station

39

<u>STATION 3 PROBLEM</u>

$$48 = 8m$$

<u>Answer to Another Station</u>

27

STATION 4 PROBLEM

$$\frac{2}{3}m - 7 = 19$$

Answer to Another Station

12

STATION 5 PROBLEM

Mr. Smith was so pleased with the amount of homework his students completed that he bought sandwiches, fries, and drinks for the whole class. The total cost was $100. The sodas and fries cost $73.27. If the sandwiches cost $0.99 each, how many sandwiches did he buy?

Answer to Another Station

7

STATION 6 PROBLEM

$$-12 = \frac{1}{2} + \frac{5}{8}m$$

Answer to Another Station

14

STATION 7 PROBLEM

$$9 = 16 - m$$

Answer to Another Station

5

STATION 8 PROBLEM

Friendly's has a special offer on milkshakes right now. If you buy their limited edition glass for \$5.80, then you can refill it (with a milkshake) for \$1.25 anytime during the month. If you have \$23.30, how many milkshakes can you buy this month?

Answer to Another Station

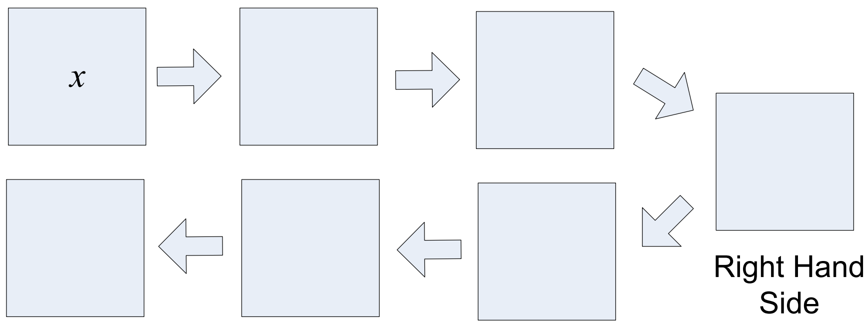
6

Student Worksheet for "Speed Mathing"

Directions: Sit at any seat you like. Begin the first problem. **Make sure you show the work for the problem in the correct area of this worksheet.** (If you start at problem 4, make sure you show the work under problem 4 on this worksheet.) You can work with the person sitting across from you, if there is a person sitting across from you, but you must each show your work on the paper below. When the timer goes off, move to your right and begin that problem. Continue repeating until you've completed all the problems.

Objectives: Collaborate with your "partner" to solve each problem before the timer goes off.

Problem 1:



Check:

Problem 2:

Define the variable: Write an equation:

Solve the equation:

Answer the problem in a full sentence: ____

Problem 3: Draw tiles:

Describe how to solve the equation using the tiles: ____

____

Solution: Check using substitution:

Problem 4:

Solve:

Check using substitution:

Problem 5:

Define the variable:

Write an equation:

Solve the equation:

Answer the problem in a full sentence: ____

Problem 6:

Solve:

Check using substitution:

Problem 7:

Solve:

Check using substitution:

Problem 8:

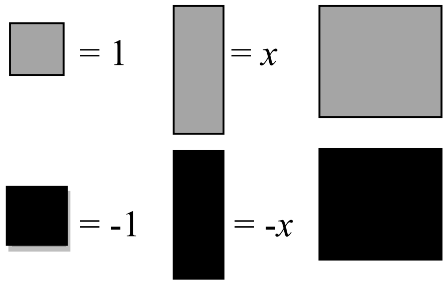
Define the variable:

Write an equation:

Solve the equation:

Answer the question in a complete sentence:

Combining Like Terms with Algebra Tiles

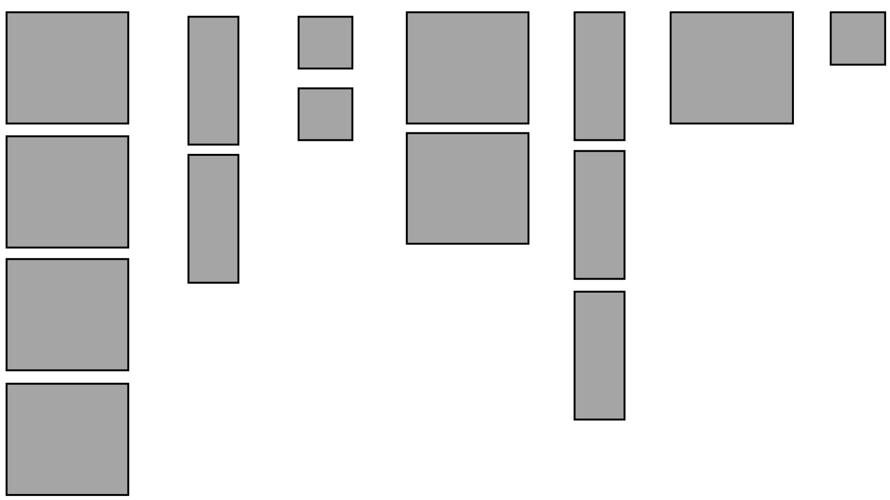
Key	Rule to Remember
 = 1 = x = x^2 = -1 = -x = -x^2	Like terms in an equation have the same variable and exponent. They do not need to have the same coefficient.

Algebra tiles can be used to model algebraic expressions and simplify expressions by combining like terms. Carefully look at the example below. Each term in Example 1 is positive.

Example 1: Simplify $4x^2 + 2x + 2 + 2x^2 + 3x + x^2 + 1$ using algebra tiles.

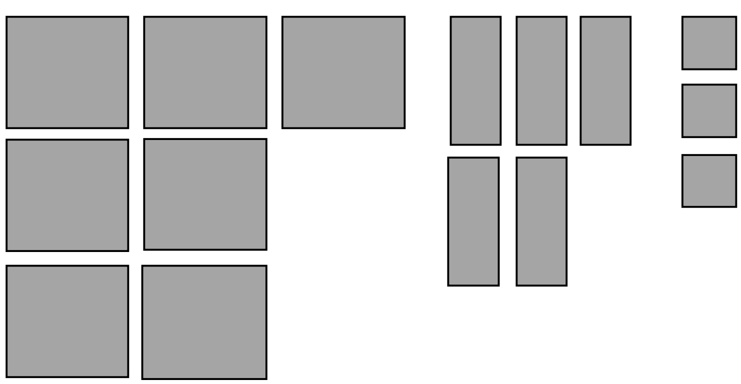
First represent each term with algebra tiles.

$4x^2$ $2x$ 2 $2x^2$ $3x$ x^2 1



Then group the similar tiles together and state the result.

$7x^2$ $5x$ 3



The resulting expression is $7x^2 + 5x + 3$.

1. Use algebra tiles to model the expression and combine like terms.

a) $4x + 1 + x + 5$

b) $2 + 3x + 5x + 4x + 1$

c) $2 + x^2 + 3x + 2x^2 + 2$

d) $2x + 3x^2 + 3x + 2x^2 + 2 + x^2 + x^2 + 1$

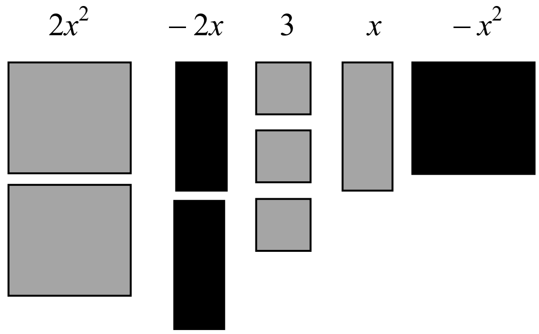
e) $x^2 + 2x + x^2 + 1 + x^2 + x + 1 + x^2 + 2$

f) $x^2 + 2x + x + 2x^2 + x^2 + 3 + 3x + 1 + x^2 + 3x^2$

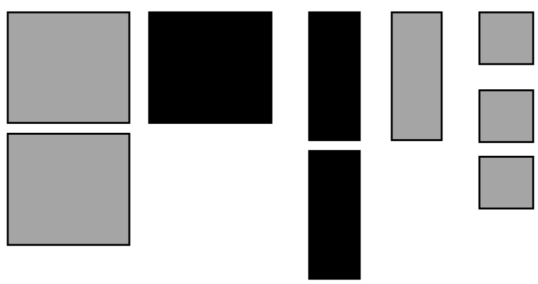
Algebra tiles can also be used when an expression contains negative terms. Black tiles represent the negative terms. Negative terms are terms that are being subtracted.

Example 2: Simplify $2x^2 - 2x + 3 + x - x^2$ using algebra tiles.

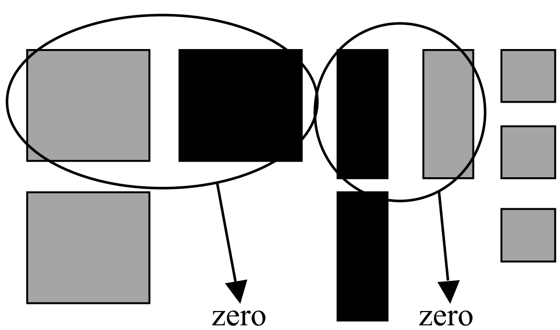
First represent each term with algebra tiles.



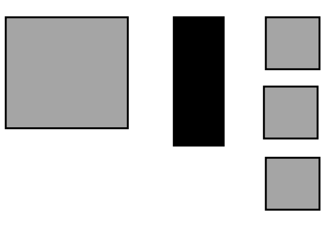
Group the similar sized tiles together.



Remove pairs that equal zero.



Show the result.



The resulting expression is $x^2 - x + 3$.

2. Use algebra tiles to model the expression and combine like terms.

a) $3x - 2 - 2x + 4$

b) $x + 3 - 2x - 2 - x$

c) $1 - x^2 + 2x + 2x^2 - 2$

d) $x^2 - 2x - 2x^2 + x^2 - 3 + 3x + 1$

Solving Equations with Variables on Both Sides

1. You are looking for a dog spa to care for your dog. Each time you visit a spa, you ask them to give your dog a bath. There are two companies you are considering. Golden Dog Care charges $20 for each visit plus $5 for each bath. Super Dog Delight charges $15 for each visit and a $60 fee for an unlimited number of baths. You need to know how many visits will make the two companies charge the same amount.

   a) Write an expression for the amount that Golden Dog Care will charge for *n* visits.

   b) Write an expression for the amount that Super Dog Delight will charge for *n* visits.

   c) Write an equation which determines when these companies charge the same amount.

   d) We can solve this equation in three different ways. Let's first use a **table**. Complete the table below.

Number of Visits	Golden Dog Care	Super Dog Delight
0		
1		
2		
3		
4		
5		
6		

   e) What are some disadvantages to using a table to find the solution of an equation?

f) Now solve the equation using the **properties of equality**. List your steps, and show all of your work. Check your solution.

g) Now solve the equation by making a **graph** on your graphing calculator. Set your graphing window using the values in the window below:

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=2
 Ymin=0
 Ymax=500
 Yscl=50
↓Xres=1
```

In the Y= menu, enter the left side expression into Y_1 and the right side expression into Y_2. Press GRAPH and find the point where the two lines intersect.

h) What are some disadvantages to using a graph to find the solution?

2. Dennis is collecting aluminum cans to raise funds for a local animal shelter. He needs to determine whether he should return the cans to the local supermarket or return the cans to the recycling center. If he brings the cans to the supermarket, he receives 5 cents per can. If he brings the cans to the recycling center, he receives 6 cents per can but is forced to pay a $15 recycling fee. How many aluminum cans would he have to return to receive the same amount of money at the supermarket and the recycling center? Write an equation that describes this problem, and then solve it.

3. Create your own problem by filling in the blanks below.

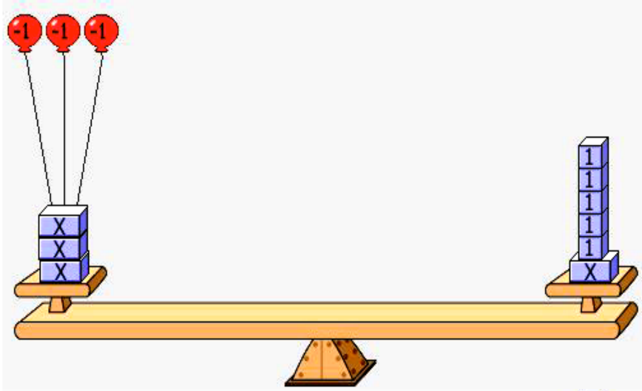
   A **membership** to a rock-climbing gym allows you to climb as much as you want for a fee of $_____ but you also pay $_____ per day for equipment rental. **Nonmembers** pay $_____ per day to use the gym and $_____ per day for equipment rental**.** Find the *number of days* in which the total cost for the members and nonmembers *are the same*.

   Write an equation that solves the problem you created above. Then solve the equation and check your solution.

Solving Equations with Balance Scales

To explore equations with variable terms on both sides, go to the Algebra Balance Scales Applet on the National Library of Virtual Manipulative (NLVM) website at:
http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html

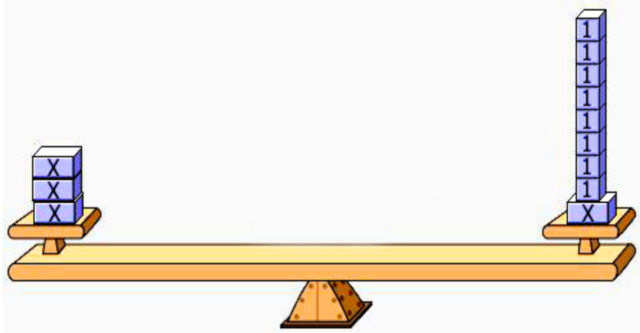
Here is an example of an equation being solved with the applet. The rectangles represent x, the squares represent one, and the red balloons represent negative one.



The equation is:

$$3x - 3 = x + 5$$

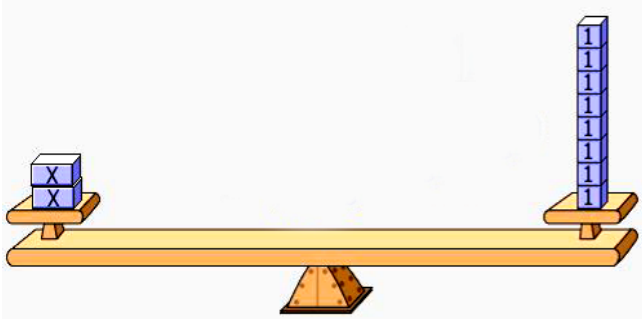
Step 1: Add 3 to both sides.



The equation is now:

$$3x = x + 8$$

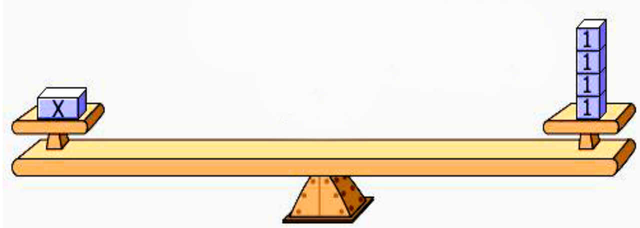
Step 2: Subtract x from both sides.



The equation is now:

$$2x = 8$$

Step 3: Divide both sides by 2.
(Remove half of each side.)



The equation is now:

$$x = 4$$

This is the solution.

As you have seen, sometimes you will have variable terms on both sides of the equation. To solve these equations, we must get all the variable terms on one side, get all the constant terms on the other side, and then combine like terms.

Solve each equation below. Show your work at each step. You may use Algebra Balance Scales to model and solve the equations.

1. $6w = 2w - 8$

2. $5 - 3m = 2m$

3. $k + 4k = 3k + 6$

4. $7a - 9 = 8a - 4$

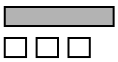
5. $-y + 8 = 3y + 4$

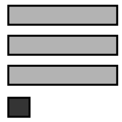
6. $2 + 3u + 1 = 4u + 3$

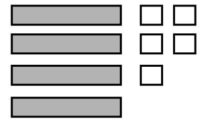
Distributive Property with Algebra Tiles

KEY: $= x$ $= 1$ $= -1$

Write the expression represented by each group of tiles.

1. 

2. 

3. 

Represent each expression using tiles. Draw your tiles below each expression.

4. $2x + 4$

5. $3x - 1$

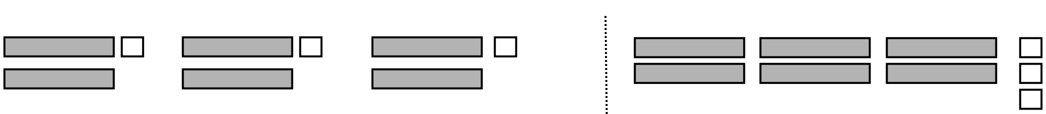
6. $x - 4$

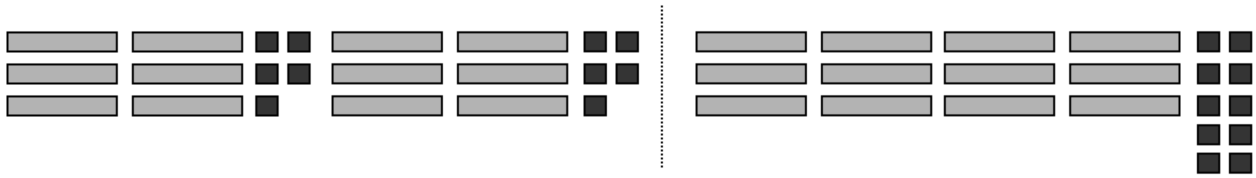
The expression $2(2x + 3)$ indicates two groups of tiles as shown below on the left. You can combine like terms to simplify the expression and you get what is shown below on the right.

$\mathbf{2(2x + 3)} = \quad 2x + 3 \ + \ 2x + 3 \quad = \quad 4x + 6$

The tiles show that the product of 2 and $(2x + 3)$ is $4x + 6$. This illustrates the **Distributive Property**. The model above represents the equation $2(2x + 3) = 4x + 6$.

Write an equation for each model. (Use the example above as a guide.)

7. 

8. 

9. 

Use tiles to represent the expression. Then rewrite the expression without parenthesis.

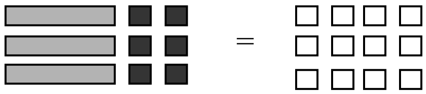
10. $3(x+1)$

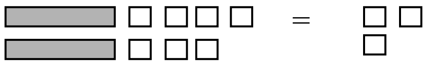
11. $4(2x-3)$

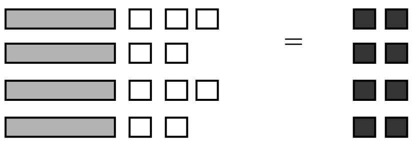
12. $2(3x-1)$

13. Does $4(2x-5)$ equal $8x-5$ or $8x-20$? Explain.

Write an equation to model the picture.

14. 

15. 

16. 

Draw a picture to model the equation.

17. $4x - 7 = 1$

18. $2(3x + 1) = 8$

19. $3(2x - 3) = 3$

In problems 20 – 23, decide if the distributive property was applied correctly. Explain your answer.

20. $4(2x + 1) = 8x + 1$ **YES** or **NO**

21. $5(3x - 2) = 15x + 10$ **YES** or **NO**

22. $7(3x - 1) = 21x - 7$ **YES** or **NO**

23. $3(2 - 6x) = -18x + 6$ **YES** or **NO**

24. Suppose one of our classmates was absent from class today. She will need to know what the distributive property means. Look over your work on this activity and briefly describe the distributive property below.

AWalk-a-Thon

Raul's school decided to participate in a walk-a-thon to raise money for a local charity. His homework last night was to get pledges and bring them back to class. Three of Raul's aunts said they would each pledge \$4.50 for his participation and then \$1.50 for every mile that Raul walked. His classmate Casey got her grandfather to pledge \$200 for the walk-a-thon. His friend Thanoj got his mother and sister to each pledge \$8.00 for every mile that he completed.

1. If we let m = miles walked, then the expression: $3(4.50 + 1.50m)$ describes the amount of money pledged to Raul. Briefly explain why this expression models his pledge amount.

2. Write expressions for Casey's and Thanoj's pledges.

3. In your opinion, which of the three students received the "best" pledge? Briefly justify your answer.

4. Anyone who earns more than \$100 receives an iTunes gift card. How many miles would Raul need to walk to earn the gift card? Explain how your answer.

5. How many miles would Thanoj need to walk to earn the gift card? Does either Raul or Thanoj have a realistic chance?

6. How many miles would Raul need to walk to collect the same amount in pledges as Casey? Briefly explain your strategy.

7. How many miles would Thanoj and Raul have to walk to both earn the same amount of money?

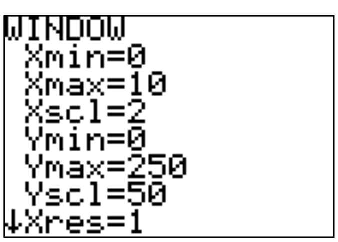
8. The graphing calculator can also be used to explore the three students' pledges. Put Raul's pledge expression into Y_1, Casey's into Y_2, and Thanoj's into Y_3. Use the table feature of the graphing calculator to solve problems 3 – 7. Copy values from the tables in the graphing calculator into the tables below. Then explain how the values in the tables may be used to answer these problems.

Raul	
x	***y***
0	
1	
2	
3	
4	
5	

Casey	
x	***y***
0	
1	
2	
3	
4	
5	

Thanoj	
x	***y***
0	
1	
2	
3	
4	
5	

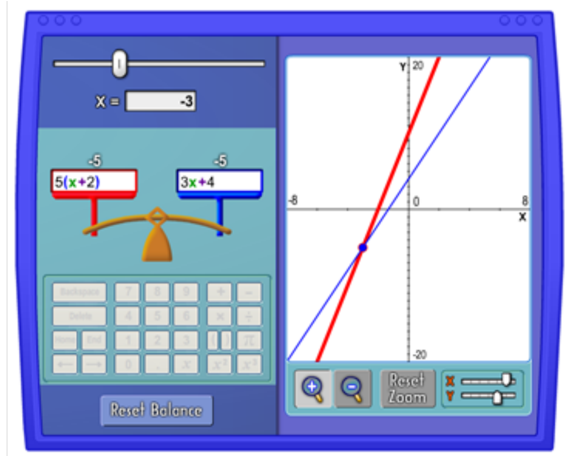
9. Graph the three equations together in the same window. Use the window settings below. Does the graph verify your predictions? How is using a graph similar to using a table? How is it different?



Epic Fail, Epic Win

1. Solve the following equation.

$$5(x+2) = 3x + 4$$



2. Using the Pan Balance Applet at NCTM Illuminations, I entered one side of the equation in the left pan and the other side in the right pan. What does the image above tell you about the value(s) of x that makes the pans balance?

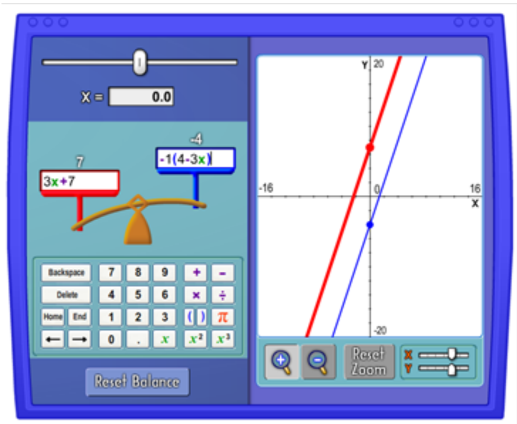
3. Solve the following equations.

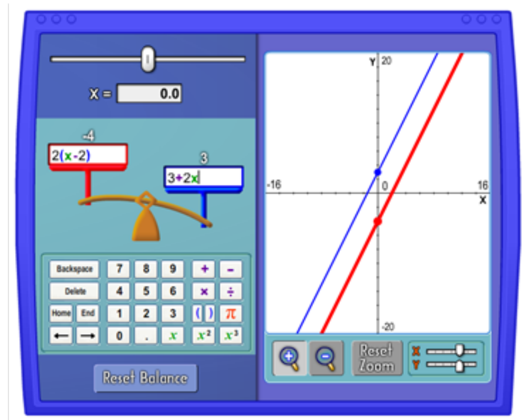
(a) $2(x-2) = 3 + 2x$

(b) $3x + 7 = -(4 - 3x)$

4. What happened? We get false statements and cannot find a real number that makes the statements true.

5. **EPIC FAIL!**
What do the following images tell you about the value(s) of *x* that make the pans balance?

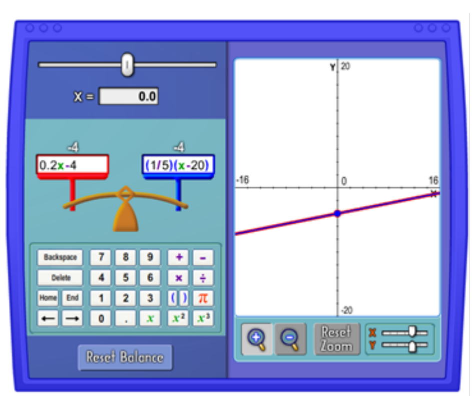


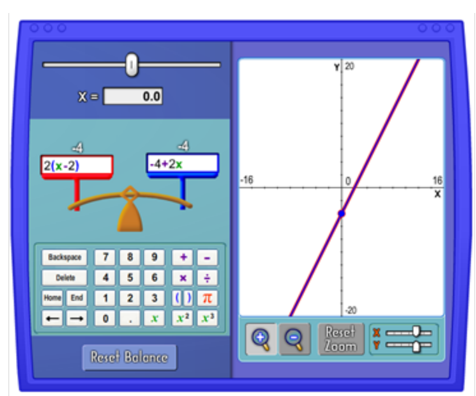


6. Solve the following equations.
(a) $2(x-2) = -4 + 2x$ (b) $0.2x - 4 = \frac{1}{5}(x - 20)$

7. What happened? We get true statements. The expressions on both sides are identical.

EPIC WIN!
8. What do the following images tell you about the value(s) of *x* that make the pans balance?





Vocabulary:
Equations where **no** value of *x* can make them true are called contradictions (EPIC FAILS!), and equations where **any** value of *x* can make them true are called identities (EPIC WINS!).

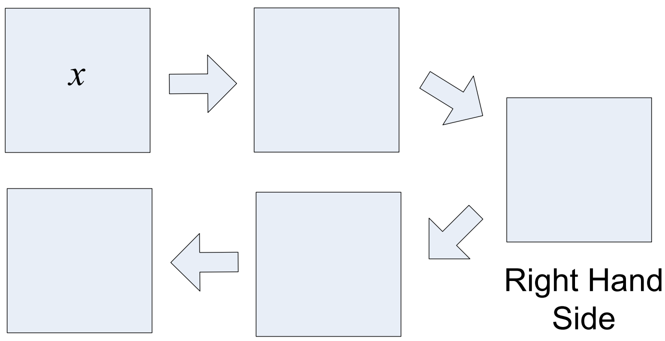
9. Write down everything you see about the equations, the graphs and the solutions for the above problems.

Transforming Literal Equations with Flowcharts

You will now use flowcharts to solve for variables in literal equations. Apply the corresponding steps to the equation on the right side.

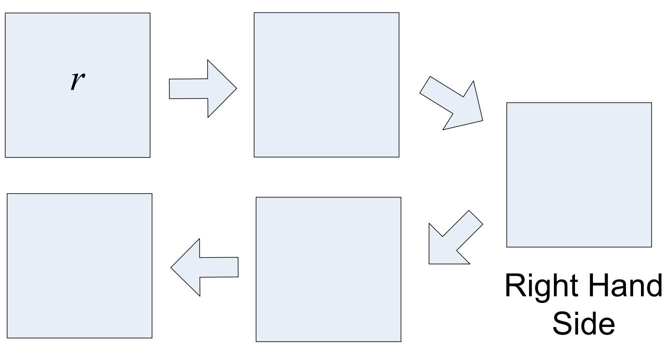
1. Solve $24 + x = z$ for x.

$24 + x = z$



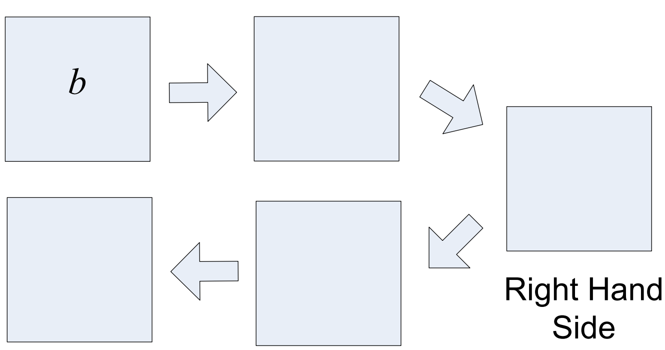
2. Solve $2\pi r = C$ for r.

$2\pi r = C$

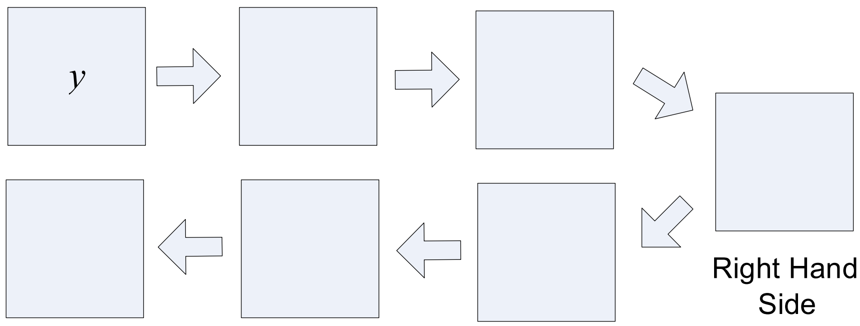


3. Solve $\frac{bh}{2} = A$ for b.

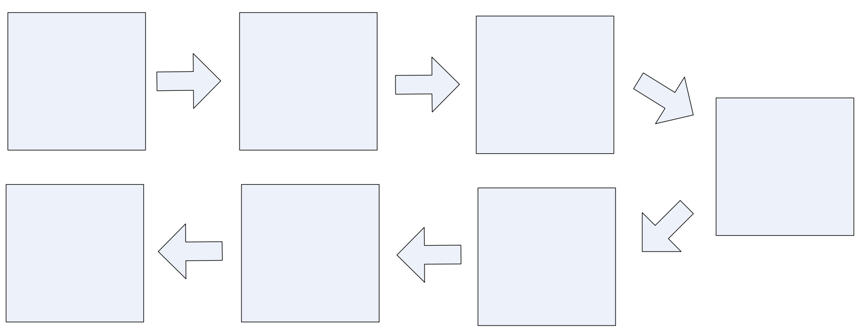
$\frac{bh}{2} = A$



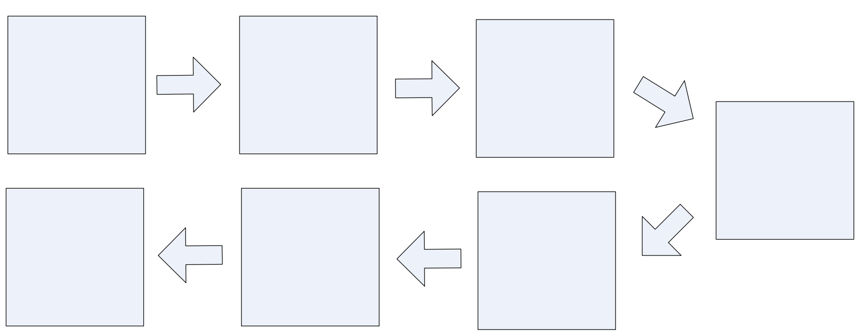
4. Solve $8x + 2y = 2$ for y. $8x + 2y = 2$



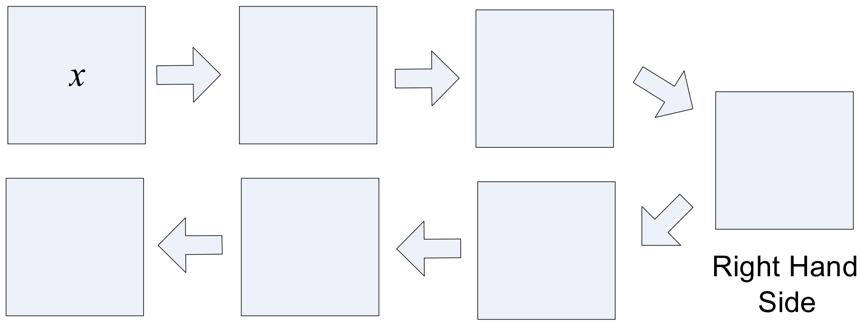
5. Solve $A + B + C = 180$ for A. $A + B + C = 180$



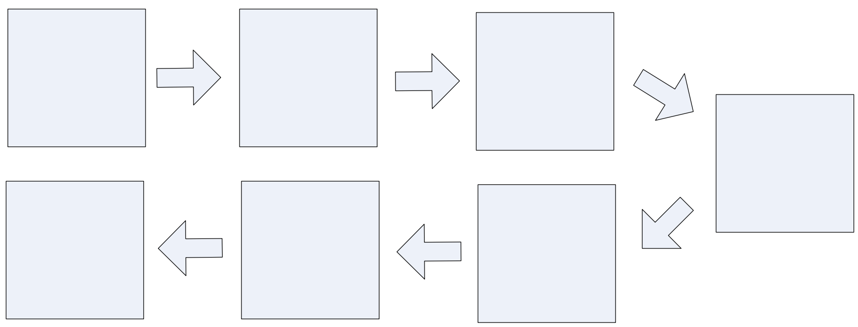
6. Solve $prt = I$ for t. $prt = I$



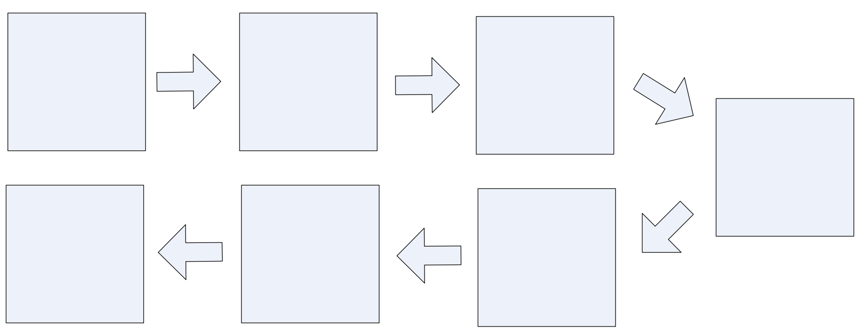
7. Solve $2x + 5y = 20$ for x. $2x + 5y = 20$



8. Solve $12x - 3y = -6$ for y. $12x - 3y = -6$



9. Solve $x - 4y = 28$ for y. $x - 4y = 28$



Unit 2: Investigation 5 (2 days)

Formulas and Literal Equations

CCSS: A-CED 4, A-REI 3

Overview

Students apply the principles of solving equations to literal equations in order to change the subject of an equation or formula.

Assessment Activities

Evidence of Success: What students will be able to do?
Change the subject of a formula in a literal equation and explain why one would want to change the subject of a formula.

Assessment Tools: How they will show what they know.

- **Exit Slip 2.5** asks students to change the subject of a formula.
- **Journal Entry 1** has students explain what it means to solve for a particular variable.
- **Journal Entry 2** has students show that they understand the meaning of "literal equation."

Launch Notes

You may pose the following problem to the class: "Tanya is selling shoes at a shoe store. She finds out that shoe size and foot length are related by the formula $S = 3F - 24$ where S represents the shoe size and F represents the length of a foot, in inches. Tanya wants to know how large the feet of her customers are on a certain day."

First ask students to complete a table, similar to below, using the given formula.

S	*F*
5	
6	
9	
11	

Next ask students to use the formula $F = \frac{S+24}{3}$ to find values for F (using the same S values from the table). Ask students what they notice about the two sets of values. Ask them which method is easier to use to find F values. Students should see the benefit in having an equation in which the variable of interest is the subject of the equation.

Closure Notes

Use the group activity to assess students' ability to manipulate literal equations. Ask students to summarize what they learned about solving literal equations.

Teaching Strategies

I. Use the example from the launch notes to introduce the topic of formulas. The formula $S = 3F - 24$ gives us the shoe size when the length of the foot (in inches) is given. Inform students that S is the *subject* of the formula. We can find values of the subject by substituting in for the other variable or variables. Since we know how to solve equations we can also find values of the other variables in the formula when the subject is known. For example, if we know a value of S, we can solve for the corresponding value of F (as we was done to fill in the table).

However, we can also rearrange the formula to make a different variable the subject. We can do that with the formula $S = 3F - 24$ by applying the properties of equality we use to solve equations (add 24 on both sides, and then divide both sides by 3). This results in the formula $F = \frac{S+24}{3}$ given above. Mention that this formula would be useful if we had a large number of shoe size values and wanted to find their corresponding foot lengths.

II. Introduce the term *literal equation*, which may be defined as an equation with more than one variable. (The word "literal" refers to "letter'). In the example above, $S = 3F - 24$ is as a literal equation. It is *solved for F in terms of S.* "Changing the subject of the formula" and "solving a literal equation" are two ways to describe the process of solving for a different variable.

III. In **Activity 2.5.1 Literal Equations**, students explore a variety of literal equations. Students manipulate formulas for temperature, surface area, sum of angles in a regular polygon, cricket chirps based on temperature, and simple interest. You may want to consult with science teachers about what formulas students are using in their science classes. These would provide additional meaningful examples for changing the subject of a formula because students will be using them in more than one discipline.

IV. In **Activity 2.5.2 More Literal Equations**, students solve for variables using formulas which they have previously used. From their previous, studies should recognize the geometrical formulas in the activity. Many of these formulas are used on the Connecticut Academic Performance Test.

V. In **Activity 2.5.3 Literal Equations with Flowcharts**, students use flowcharts to transform literal equations. The flowcharts require students to identify the operations *being done* on the variable they wish to solve for, and then use flowcharts to identify the inverse operations which must be performed to make the variable the subject. This activity utilizes prior knowledge and use of flowcharts developed in Activity 2.2.1.

Differentiated Instruction (For Learners Needing More Help)

Flowcharts (**Activity 2.5.3**) provide visual cues and are recommended for students who have difficulty identifying the appropriate steps.

VI. **Activity 2.5.4 Green Problems** provides students an opportunity to explore equations which relate to environmental and conservational issues.

Differentiated Instruction (Enrichment)

The green problems (**Activity 2.5.4**) require students to research energy costs and information about home appliance energy efficiency ratings and insulation energy efficiency ratings.

VII. You can also motivate the need to change the subject of a formula by having students write simple programs for the TI-83/84 calculator to evaluate a formula on their calculators. For example, the program below on the left side finds the perimeter of a rectangle given its length and the width. The program below on the right side finds width of a rectangle given the rectangle's length and perimeter. Students may then solve for W and write a program to find the width.

PROGRAM PERIM

```
:Prompt L
:Prompt W
:2L + 2W → P
:Disp "P ="
:Disp P
```

PROGRAM WIDTH

```
:Prompt L
:Prompt P
:(P – 2L)/2 →  W
:Disp "W ="
:Disp W
```

VIII. Give students equations in two variables, *x* and *y*, and ask them to solve for *y* in terms of *x*. This is a skill that will be used in Unit 4 to transform linear equations from one form to another and in Unit 6 to solve systems of equations by substitution.

For example, given the equation $12x + 4y = 20$, we can solve for *y* when *x* is known, say $x = 2$, or when *x* is unknown. Either way we will implement the similar steps. The left side shows the process when $x = 2$, and the right side shows the process when *x* is unknown.

$$\begin{array}{rcl} 24 + 4y &=& 20 \\ -24 & & -24 \\ \hline 4y &=& -4 \end{array}$$

$$\begin{array}{rcl} 12x + 4y &=& 20 \\ -12x & & -12x \\ \hline 4y &=& 20 - 12x \end{array}$$

$$\frac{4y}{4} = \frac{-4}{4} \qquad \frac{4y}{4} = \frac{20 - 12x}{4}$$
$$y = -1 \qquad y = 5 - 3x$$

You may use a power point designed by Skip Tyler, a teacher at Varina High School in Henrico County, Virginia. It may be found at http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-5.html
The power point helps students see that the same steps used to solve an equation in one variable can be used to solve a literal equation.

Differentiated Instruction (Enrichment)

Put the following equations on the board and ask students if these are in fact the same formula. They must justify their answer.

$$s = \frac{n}{n+1} \qquad \frac{s}{s-1} = n$$

Group Activity

Pick four problems involving literal equations, possibly from **Activity 2.5.1** or **Activity 2.5.2**, and place them in stations around the room. Break students up into groups of four. Within each group, assign each student a number from 1 to 4. Students then go to the station that has their assigned number. There they work for three to five minutes individually to solve the problem. They may then work with others at their station to find the solution. Once they have the solution they should discuss it so that each of them can explain the solution process.

Then have students return to the original group as an "expert" on the problem they just solved. Each student takes 3 minutes to explain the solution to his or her problem with the other students writing the problems and solutions in their math notebooks. This will take about 25 to 30 minutes. Encourage others in the group to ask questions for clarification.

You may then ask students to generalize what they have learned about solving literal equations in a whole class discussion to provide closure to this investigation.

Journal Entry

1. Given an equation that has x as a variable, what does "solve for x" mean?
2. Explain why $r = \frac{6.27}{2\pi}$ is not a literal equation.

Resources and Materials

- **Activity 2.5.1** Literal Equations
- **Activity 2.5.2** More Literal Equations
- **Activity 2.5.3** Literal Equations with Flowcharts
- **Activity 2.5.4** Green Problems
- **Exit Slip 2.5** Cricket Chirps
- Graphing calculators
- Student Journals
- Link to the instructions for writing simple programs for the TI-83 or TI-84 graphing calculators http://education.ti.com/.
- Link to power point for solving literal equations http://teachers.henrico.k12.va.us/math/hcpsalgebra1/module3-5.html

Unit 2: Investigation 6 (4 Days)

Linear Inequalities

CCSS: A-CED 1, A-REI 3

Overview

Students write linear inequalities that model real world scenarios and solve multi-step linear inequalities.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
Write and solve linear inequalities in context, justify why the inequality symbol is reversed when multiplying or dividing by a negative number, and solve multi-step linear inequalities.

Assessment Strategies: How Will They Show What They Know?

- **Exit Slip 2.6.1** asks students solve multi-step linear inequalities.
- **Exit Slip 2.6.2** asks students to write a linear inequality to model a situation and then solve the inequality and interpret the solution.
- **Journal Entry** asks students to explain when and why they reverse an inequality symbol when solving an inequality.

Launch Notes

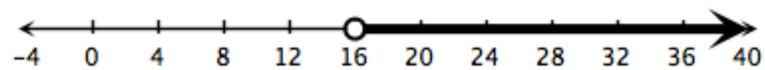
Introduce students to several real-life examples of linear inequalities by showing them situations which involve maximum and minimum values. **Activity 2.6.1 Representing Inequalities** contains several examples and can be used for this purpose. The first example in this activity is, "To serve as president of the United States you must be at least 35 years old." Students will recognize that this is an example of an inequality and that it can be represented by a set of numbers on a number line. At this time you may lead a discussion about the nature of continuous variables (when $x \geq 35$, x isn't only satisfied by integers, but all real numbers greater than or equal to 35).

Closure Notes

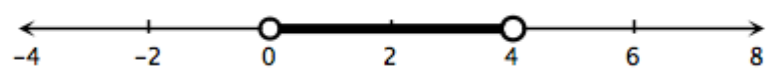
Students should recognize the difference between an equation and an inequality. You may lead a class discussion on their similarities and difference between linear equations and linear inequalities.

Teaching Strategies

I. **Activity 2.6.1 Representing Inequalities** uses real world examples to show students how to represent inequalities on a number line. As students complete the activity, make sure that they understand the nature of continuous variables. A common misconception among students is that a solution set only contains integers. For example, if a student is asked to graph $x > 16$, many will start at 17. You may ask students questions like "Is 16.1 greater than 16? Where is it located on the number line? How can we represent this number?" Students should recognize that this is a situation which warrants an open circle as shown below.



Here is another example. "To get on this ride, a child must be less than 4 feet tall to play here." The set of numbers described in this statement is all numbers between 0 and 4. If students are only considering integers between 0 and 4, ask them, "what about a child who is 3 feet 11 inches tall? How can we represent that 3 feet 11 inches is included?"



This example may be used to show how open circles represent endpoints that are not in the desired set. Make sure students understand why both 0 and 4 must be excluded from the solution set and draw the number line representation.

You may also mention that in some real world situations all real numbers may not be appropriate. Suppose you have $100 to buy books, each book costs $13.85, and you want to find out how many books you can buy? Solutions of this problem only include positive integers and zero. Negative numbers will not make any sense. Linear inequalities in one variable provide us an opportunity to talk about the different sets of numbers and why we select one set over another in a given contextual situation.

II. In **Activity 2.6.2 Equations and Inequalities**, students examine the distinction between the solution of an equation and the solution of an inequality, determine whether numbers are included in a solution set, and learn about how solutions are represented on a number line. Students also construct written descriptions of solution sets and identify terminology and phrases associated with inequality symbols.

III. Once students are familiar with how to represent an inequality on a number line, they are ready to solve multi-step inequalities. The rules for solving equations will be used, with one important exception: when multiplying or dividing both sides of an inequality by a negative number, we reverse the direction of the inequality symbol. Students may refer to this as "flipping" the inequality symbol. This concept is developed in **Activity 2.6.3 When Do We Flip It?** In this activity, students discover that adding a positive or negative number to an inequality does not *affect* the inequality symbol, but multiplying or dividing by a negative number does lead to a reversed inequality symbol. The activity culminates with students applying this property to solve multi-step linear inequalities and graph solutions.

IV. The concept of reversing the inequality symbol can be developed by taking an inequality such as 2 < 4, graphing 2 and 4 on a number line, noting that 2 lies to the left of 4, then multiplying 2 and 4 by –1, hence generating their *opposites*, and then having students graph the two opposites on a number line. Students will see that the order is not retained. This process can be repeated with additional inequalities like –10 < 9, and – 4 < 7. Help students understand that when we multiply or divide by a negative quantity we are now trying to relate the opposites of the original numbers (or multiples of the numbers). The opposites are related in reverse as a consequence of the definition of an opposite.

V. The TI-84 program LINEQUA allows students to represent inequalities by tables and graphs on their graphing calculators. The program is supported by teacher notes and a student activity sheet. In order for students to use the program LINEQUA, it must be downloaded into their graphing calculators. (Program and website can be found at http://education.ti.com/xchange/US/Math/AlgebraI/8773/LinearInequalities_Teacher.doc

VI. **Activity 2.6.4 Working with Inequalities** provides students additional practice solving one- and two-step inequalities. Students express inequalities in words and on a number line, and identify numbers in a solution set and check whether the numbers they chose satisfies the inequality. Students also identify inequalities given solution sets graphed on number lines.

Differentiated Instruction (For Learners Needing More Help)

Some students have difficulty recognizing that an inequality like $8 > x$ is equivalent to the inequality $x < 8$. Questions 15 and 16 in **Activity 2.6.2** specifically address this issue. If confusion on this score persists, you may teach students to always get the variable on the left side. This will help in graphing the solution set once the inequality is solved. A mnemonic that helps is to use the direction of the inequality symbol to show which side of the end point to shade. For $x > 8$ we shade to the right of 8 (the direction the symbol is pointing). For $x < 8$ we shade to the left. If you teach this strategy, emphasize that the variable must be on the left for this to work. **Activity 2.6.4** provides opportunities for students to rewrite expressions.

VII. **Activity 2.6.5 Practice Solving Inequalities** has a variety of multi-step linear inequalities. Students get practice applying the distributive property and combining like terms.

VIII. **Activity 2.6.6 Putting It All Together** designates problems for students to work on individually, problems for students to work on as a class, and problems for the instructor to work on to model the solution process. The activity starts with two-step inequalities, and then moves to contextual problems, and then multi-step inequalities. The inequalities include an identity and contradiction.

Group Activity

In **Activity 2.6.7 Passing Linear Inequalities** students collaboratively solve problems by "passing the problem" within a group. Divide students into groups of size three and assign each student in the group a different problem set. They should each do the first step in solving the inequality and then pass the problem to another student in their group. That student will do the second step and then pass the problem to the third student. The third student will graph the solution on the number line. The problem is then passed back to the original student to check the work. This process is then repeated for two additional problems. The last problem is a word problem. In this problem, the first student should write the inequality, the second student should solve the inequality, and the final student should graph it *and* write the solution in a sentence.

Journal Prompt

Explain when you have to "flip" the inequality symbol and why you have to "flip" it.

IX. In **Activity 2.6.8 Inequalities in the Real World**, students solve inequalities arising from real world situations. Each problem requires that they first write an inequality to model a situation. The final problem in the activity focuses on the cost of a cellular phone family plan based on the number of phone lines. As an extension to this problem, students could compare cellular plans and set up inequalities to determine when a family plan for several people is cheaper than individual phone plans for each family member.

You may introduce the value of modeling with inequalities by presenting the following school fundraising scenario. "You have been selected to pick out a T-shirt that everyone should wear at the annual pep rally. The student council has set aside $6,000 to purchase the shirts. Go online (to a website like www.prepsportswear.com) and look up the cost of a shirt that you like. If the online shipping cost is $14, how many of the shirts that you selected can the student council buy?"

At this point, you might want to remind students that schools are tax-exempt. Students may suggest setting up an equation at this point. You can have a discussion with the students about how you don't have to spend all $6,000 dollars but you definitely can't spend more than that. If the T-shirts are $21.96 each, then the inequality would be $\$21.96x + 14.00 \leq \$6000$, where x is the number of T-shirts. Ask students to explain how they would solve for x and ask them show their steps and check their answer. In this case, x is about 272.6. Facilitate a class discussion about this answer and why the solution shows that they can buy up to 272 T-shirts. Have them discuss why rounding down in this case is necessary versus rounding up to the closer value of 273 shirts. For more practice, students could solve similar problems related to purchasing school uniforms, school equipment, or other school objects of interest.

X. You may introduce problems modeled by inequalities with variables on both sides. For example, in **Activity 2.3.7 Comparing Cab Fares**, students determined the distance for which two companies charged the same amount. You can modify these problems to find when one company is less expensive or more expensive than another company.

Differentiated Instruction (Enrichment)

Have each student construct a contextual problem that can be solved with a linear inequality. Ask them to solve it and write the answer on a separate piece of paper. Collect all of the problems (without answers) and shuffle them. Then, have students select a problem from the deck and attempt to solve it. Answers can be distributed at the end of the activity, and the problems can be discussed.

Resources and Materials

- **Activity 2.6.1** Representing Inequalities
- **Activity 2.6.2** Equations and Inequalities
- **Activity 2.6.3** When Do We Flip It?
- **Activity 2.6.4** Working with Inequalities
- **Activity 2.6.5** Practice Solving Inequalities
- **Activity 2.6.6** Putting It All Together
- **Activity 2.6.7** Passing Linear Inequalities
- **Activity 2.6.8** Inequalities in the Real World
- **Exit Slip 2.6.1** Solving Inequalities
- **Exit Slip 2.6.2** How Many Pizzas?
- Student Journals
- Graphing Calculators
- LINEQUA information on TI website http://education.ti.com/educationportal/sites/US/homePage/index.html
- Website that sells customized clothing www.prepsportswear.com