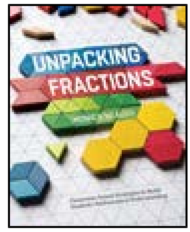
Unpacking Fractions: Classroom-Tested Strategies to Build Students' Mathematical Understanding

by Monica Neagoy



An ASCD Study Guide for *Unpacking Fractions: Classroom-Tested Strategies to Build Students' Mathematical Understanding*

This ASCD Study Guide is meant to enhance your understanding of the concepts and practical ideas presented in *Unpacking Fractions: Classroom-Tested Strategies to Build Students' Mathematical Understanding*, an ASCD book written by Monica Neagoy. You can use this study guide either with teachers if you are a professor, teacher leader, or coach, or with younger students if you are a classroom teacher. You may already have posed to yourself or a colleague many of the questions included here, and others will become evident as you read through the book. You may choose to answer the questions *before* tackling each chapter of the book or answer them as you finish the chapter. Each chapter is rich in information—content, pedagogy, and research—so take the time to digest one at a time. The study questions provided address some of the most salient ideas in the book. They do not address all of the notions covered in the book.

Introduction: The Challenge of Fractions

1. A respected mathematics educator once wrote, "Many people have a fear of mathematics. … For most of these people, their relationship with mathematics started downhill early in elementary school, right after they were introduced to fractions." You may know some children or adults whose love for mathematics ended when fractions began. List three major reasons why understanding fractions is fundamental to a student's mathematics education.
2. Identify at least three major reasons why children struggle when they begin their study of fractions. Once you are both mindful of the importance of fractions and aware of the principal reasons fractions are so difficult for students, you will be better able to help them overcome these difficulties and thrive in their learning of fractions. This book will guide you to use effective pedagogical strategies and focused problems that foster deep understanding of the important fraction concepts.

Pause and Reflect

a. What do we mean by *fraction* in elementary school? In middle or high school?

b. What are the differences between fractions and rational numbers?

c. Is every fraction a rational number? Is every rational number a fraction?

d. What is the difference between fractions and ratios?

e. Can you give a fraction that is not a ratio? Can you give a ratio that is not a fraction?

f. Is every integer a rational number? Is every rational number an integer?

g. How are natural numbers, integers, and rational numbers related?

h. Do natural numbers, integers, and rational numbers make up the entire real number system?

Chapter 1. Convey the Many Meanings of $^{a}/_{b}$

1. Consider the symbol $^{a}/_{b}$. It can have many different meanings. Brainstorm with a partner and see how many different meanings you can come up with. If it's easier, consider a specific fraction such as ¾ and answer the same question.
2. One of the first—if not the only—meanings of *fraction* that people come up with is the "part-whole meaning." Namely, that $^{a}/_{b}$ means "*a* parts out of *b* equal parts." Consider this basic interpretation of fractions that most students learn as early as 2nd or 3rd grade. Explain why, according to the Common Core State Standards for Mathematics, it is has been recommended that students in 3rd grade should interpret $^{a}/_{b}$ as "*a* parts of size $^{1}/_{b}$." What's the difference between the two explanations? Why is this difference so important?
3. When explaining the components *a* and *b* of a fraction $^{a}/_{b}$, teachers often say, "*a* is the numerator, and *b* is the denominator." (Starting in 4th grade, students learn that the fraction line means division.) While vocabulary is important, the nomenclature does not help students grasp the mathematical meaning or role of components *a* and *b*. How would you put into accessible language for children the mathematical meanings of parts *a* and *b* of any fraction $^{a}/_{b}$?
4. The first fractions students learn are the unit fractions. Fractions ½, ⅓, and ¼ are three examples of unit fractions. Why are unit fractions foundational? What tasks might you design to help students realize for themselves that "when comparing two unit fractions, the one with the larger denominator is the smallest fraction?" Regrettably, we often just tell students to remember that fact. But *teaching* is not just *telling*!

Pause and Reflect

> Claudia, a 4th grade student, once said to me, "I understand how to take three parts out of five parts to get the fraction , but I don't understand the fraction $^{7}/_{5}$. How can I take seven parts out of five parts?" What would you say and do to help Claudia find meaning in the fraction $^{7}/_{5}$?

Chapter 2. Use Visual and Tactile Models

1. Think of the visual or tactile models you have used to represent fractions, either when learning them or teaching them. Which models do you think are most effective? Are all models equivalent? Should we use one or many models to represent a fraction? Explain your answer.
2. Consider any fraction, say ¾. What are at least three different types of models or representations students should know and be able to use in the upper elementary grades?
3. Several leading documents on fraction instruction stress the importance of helping students recognize fractions as numbers, yet students in 4th through 6th grade (and even beyond) have great difficulty conceptualizing a fraction as a single number. Why do you suppose this is the case? What are some major changes in fraction instruction that

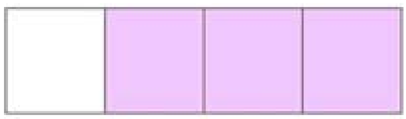
would help students see fractions as numbers before they get to middle or high school? Can you design a task that would contribute to meeting this goal?

Pause and Reflect 1

When showing the fraction ¼, we almost always (and, therefore, so do our students), begin with a whole (either continuous or discrete), partition it into four equal parts, and then designate one of those parts in some way. What students don't see enough of is teachers iterating to show ¼. Reflect on how you might model fractions ⅓ or ⅔ using two gray marbles and any number of white marbles. Since you cannot partition the marbles, think about how you might iterate to model the desired fractions.

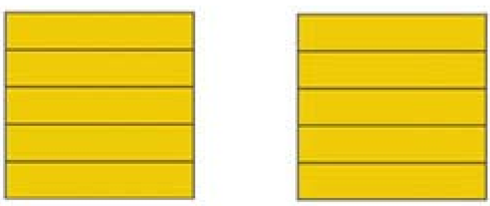
Pause and Reflect 2

Can you explain how the fraction represented by this figure could be ¾ or ¼? Could the figure represent ½ or $^{3}/_{2}$? Explain. And what about ⅓?



Chapter 3. Focus on the Unit

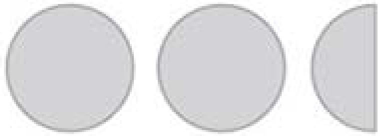
1. An essential understanding of fractions is knowing that a fraction in and of itself has no value or magnitude. A fraction represents a quantity only in relation to a whole or unit. Elementary school students don't get enough experience with a variety of units. List three of four types of units with which students should be familiar, and give an example in each case of a fraction value of your choice.
2. Could ½ be more than 1? Explain.
3. How would you enlighten a discussion between two students who disagree on the fraction depicted in the figure below? Emily says it's $^{10}/_{5}$ or 2, whereas Carlos says it's $^{10}/_{10}$ or 1? Who is correct? Could they both be? If so, explain how.



4. Can you design two tasks that will help students learn that (a) two fractions can be the same, yet the quantities they represent can be different; and (b) two fractions can be different, yet the quantities they represent can be the same?

Pause and Reflect

Consider these two and a half discs:



- Draw ½, if these discs represent the unit (i.e., find a fraction, given the unit).
- Draw $^{5}/_{6}$, if these discs represent ⅓ (i.e., find a fraction, given another fraction).
- Draw the unit if these discs represent 1¼ or $^{5}/_{4}$ (i.e., find the unit, given a fraction).

Chapter 4. Teach the Concept of Equivalence (Not Just the Rule)

1. *Equivalence* is a concept that permeates much of school mathematics. Give examples of equivalence in three other areas or levels of mathematics. In each case, explain what equivalence means.
2. Teaching equivalent fractions—regrettably—often comes down to teaching the equivalent fractions algorithm: $^{a}/_{b} = ^{(n \times a)}/_{(n \times b)}$. Reflect on how you might teach the concept of equivalence first, before introducing the algorithm. Take a simple example, such as "⅔ is equivalent to $^{4}/_{6}$." Rather than showing or telling students that "4 is two times 2, and 6 is two times 3, therefore ⅔ is equivalent to $^{4}/_{6}$," what tasks or investigations might you design to help students discover this on their own and find meaning in fraction equivalence?

Pause and Reflect

a. Describe or draw a situation in which two different fractions have identical numerical values.

b. Describe or draw a situation in which two identical fractions represent different quantities.

c. Which of these two situations model equivalent fractions? What important concept does the other situation illustrate?

d. In the first situation, we used *numerical value*. In the second situation, we used *quantity*. Is there a difference between the two? If so, describe this difference as best you can.

Chapter 5. Compare and Order Fractions Meaningfully

1. What is the difference between *additive* comparison and *multiplicative* comparison? Give an example of each.
2. We want our students to be able to reason quantitatively about fractions—to have a good sense of the size, amount, or numbers that fractions represent. Therefore, we want them to be able to compare fractions *meaningfully*. Describe three strategies for comparing fractions students should know by the time they enter 6th grade.
3. Are there are more sophisticated strategies to compare fractions? Take the example of "one-apart" fractions (as students often call them), such as $^{10}/_{11}$ and $^{11}/_{12}$. Which is greater and why?

4. Kamal understands that 8 is *two times* greater than 4. How would you build on that knowledge to help him answer the question, "*How many times* greater than 5 is 8?"

Pause and Reflect

Suppose you used Cuisenaire rods to model the comparison between two juice concentrations. Juice 1 was made with seven parts pure orange juice and nine parts water, whereas Juice 2 was made with eight parts pure orange juice and ten parts water

Knowing that most students will say, "The two juice mixtures have the same orange concentration—or are just as "orangey"—how would you guide them to see otherwise?

Chapter 6. Let Algorithms Emerge Naturally

1. What does it mean to know fraction addition (or subtraction, multiplication, or division)?
2. How would you assist and enlighten Tom in the following situation? Imagine he says, "I can come up with many stories that are modeled by the equation $24 \div 3 = 8$, but I can't come up with any that are modeled by the equation $3 \div \frac{2}{3} = 4\frac{1}{2}$ [or $3 \div \frac{2}{3} = \frac{9}{2}$]. I know how to apply the division algorithm by rote to compute the answer [$\frac{9}{2}$], but I don't see what it means to divide by a fraction."
3. An obstacle to understanding fraction multiplication and division is the ingrained belief that "Multiplication makes bigger and division makes smaller." What would you do or say to help dispel these common beliefs?
4. As students move from whole-number operations to operations with fractions, they often think they are entering a whole new world and everything they learned no longer holds. Revisit each operation to distinguish what changes and what remains unchanged as we move from whole numbers to fractions. Helping students build on what they already know about computation is key to success in fraction computation.

Pause and Reflect

For many students, knowing an operation is usually synonymous with knowing how to carry out the algorithm. Reflect for a moment on your own repertoire of contexts and real-world situations that can help students find meaning in the four fraction operations well before the teaching and learning of the algorithms for fraction computation. Do you have one or more for each operation?

Chapter 7. Connect Fractions and Decimals

1. Can you identify three or four reasons why students have difficulty with learning and

understanding decimals?

2. What is the most common model used for teaching decimals? Can you think of and describe at least three other ones we should use when teaching decimals?
3. How can we build on students' knowledge and understanding of the base-10 numeration system of whole numbers to make sense of decimals?

Pause and Reflect

> A common and unfortunate way we explain how to multiply a decimal by a power of 10 is to say, "Move the decimal point to the right *n* places," where *n* is the number of zeros in the power of 10. Reflect on what really happens, and devise a more mathematical explanation for multiplying a decimal by a power of 10. Describe representations you might use to help students visualize your explanations.

Conclusion: Moving from Rote to Reason

1. Fractions underpin many complex mathematical topics, including ratios, rates, percentages, proportions, proportionality, linearity, and slope. Pick two of these concepts and show how fractions are a necessary—but not sufficient—condition to understanding them deeply.
2. Compare fractions and ratios. In what ways are they different? In what ways are they similar? What is the difference between equivalent fractions and equivalent ratios?

Pause and Reflect

> Reflect on the journey students will travel from 2nd or 3rd grade to 8th or 9th grade—from fractions to linear functions. Can you see in your own mind how these fundamental notions build on one another and at each level become more sophisticated? Once you do, you will appreciate the importance to laying a solid foundation for fraction understanding early on.

Unpacking Fractions: Classroom-Tested Strategies to Build Students' Mathematical Understanding was written by Monica Neagoy. This 285-page, 8" x 10" book (Stock #115071; ISBN-13: 978-1-4166-2123-2) is available from ASCD for $31.16 (ASCD member) or $38.95 (nonmember). Copyright © 2017 by ASCD. To order a copy, call ASCD at 1-800-933-2723 (in Virginia 1-703-578-9600) and press 2 for the Service Center. Or buy the book from ASCD's Online Store.